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Method of Characterization of Rate-Dependent Materials

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Abstract

A category of material systems composed of crystalline particles embedded in a polymeric binder, in general referred to as a particulate composite material, has received a great deal of interest from researchers in recent years. Although the polymer binder comprises a small percentage of the unit volume of the composite, these material systems still display a significant degree of rate-dependency. In addition to the viscoelasticity of the binder component, the growth of internal damage, i.e., microcracking, is a mechanism that must be accurately characterized if the behavior of the material system is to be adequately modeled. The interaction of these two mechanisms, viscoelasticity and microcrack growth, must be modeled in a step-wise fashion. Viscoelastic deformation causes an increase in microcracking, which in turn causes a change in the viscoelastic-based strain rate.

Naturally, in order to characterize the rate-dependency of a real material system it is necessary to have appropriate data collected from tests performed on specimens of the real material. It is also necessary to be able to separate, from these data, the viscoelastic and the microcracking phenomena, and this is arguably the most difficult aspect of the characterization process.

The procedure to be outlined and demonstrated herein includes the method of separation of viscoelastic and damage response, along with determining the necessary rate-dependent damage parameters.

Keywords: plastic-bonded explosive, damage, microcracking, viscoelasticity, mechanical constitutive model.

Introduction

A plastic-bonded explosive is an example of a particulate composite material having a rate-dependency that is due to the viscoelasticity of the binder, the growth of microcracks in the binder, and the interaction of these two phenomena. A constitutive model for this type of material that is capable of accurately portraying the response of the system to thermomechanically-induced loading is obviously highly complex. The development of such a constitutive model requires 1) that the underlying theory be appropriately cast in correct numerical format, and 2) that the numerical values of the resulting model parameters be accurately determined from test data.

Typically, a wide range of rate-dependent behavior is observed for plastic-bonded explosive material systems. Thus, in developing a constitutive model for a given material system, it is necessary to undertake a complete evaluation of the available test data in order to obtain the numerical values of the model parameters. This requirement naturally leads to the need for a standard procedure to be followed in the determination of the parametric values for any particular material system displaying this type of rate-dependency.

A procedure for determining the numerical values of the model parameters in step-by-step fashion will be described through the application of the procedure to the determination of parameters for a particular material system, Mock 900-21, which is employed in mechanical testing at the Los Alamos National Laboratory (LANL).

Viscoelasticity

Rate-dependency has been traditionally thought of as being a manifestation of the viscoelasticity inherent in a material system. However, it is now an accepted fact that significant microcrack growth also occurs in composite material systems of this type when they are subjected to load. Therefore, in order to characterize the time-dependent material response, the viscoelastic behavior, as well as the accumulation of internal damage, must be quantified.

The use of a number of Maxwell model elements, connected in parallel, is generally accepted as an appropriate means of characterizing the viscoelasticity of the material system. Each Maxwell model element consists of a linear spring and a dashpot connected in series. With the viscoelastic behavior assumed to be deviatoric, the response of each Maxwell model element is characterized by a shear modulus G_n and a relaxation time τ_n where n denotes the specific Maxwell model element. This is shown schematically in Figure 1. It is necessary that one of the elements have a relaxation time of zero, i.e., a spring but no dashpot. Each Maxwell model element, excluding the spring-only element, is taken to be

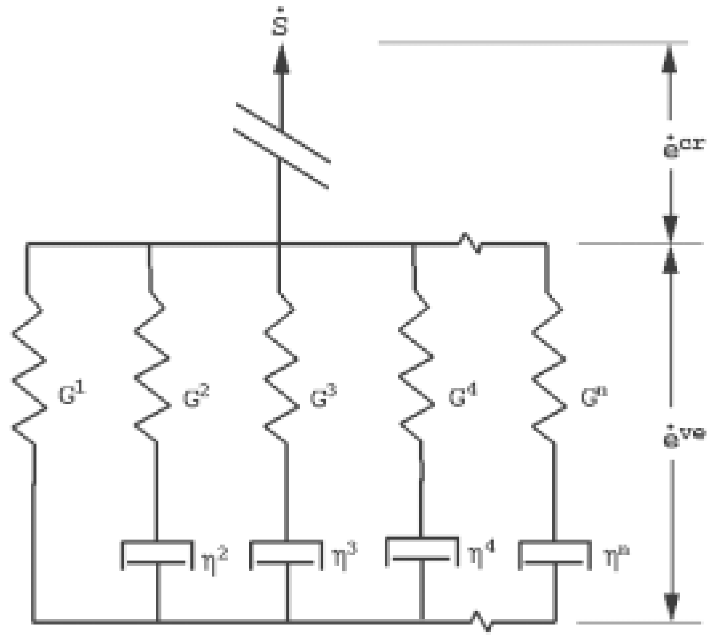
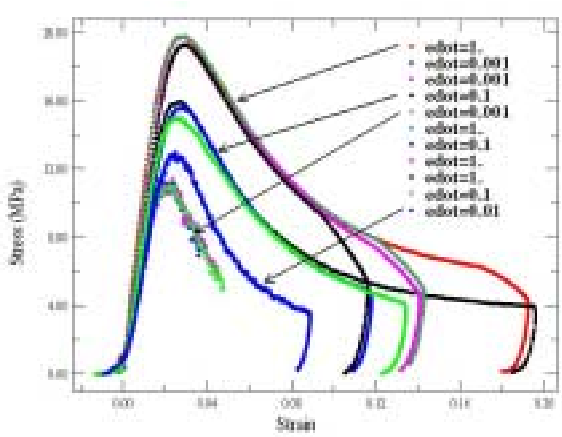


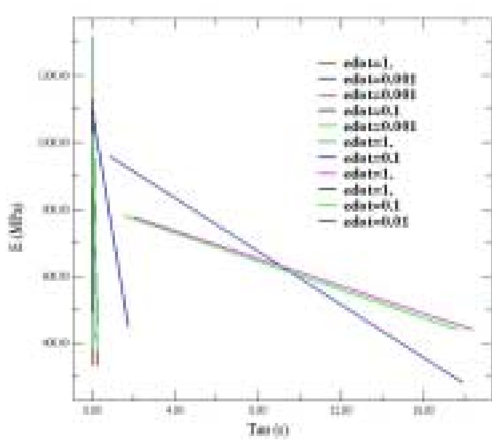
Figure 1. Generalized Maxwell model.

representative of a logarithmic decade of time. It can be seen that this linear viscoelastic model is equivalent to the widely employed Prony-series model.

In Figure 2(a) is shown the plot of stress versus strain for four sets of uniaxial constant compressive low strain-rate tests of the Mock 900-21 material. These test data are converted to corresponding Young's modulus versus relaxation time data, shown plotted in Figure 2(b). These data are obtained by curve-fitting the initial portion of the stress-strain plots (up to where the plots break over) and then employing a central-difference procedure to convert to the Young's modulus versus relaxation time data.



(a)



(b)

Figure 2. (a) Uniaxial stress-strain relationship and (b) Relationship between Young's Modulus and relaxation time at different strain rates for Mock 900-21.

It is necessary to have four or five different sets of data, each for a given strain rate, in order to obtain an accurate set of the appropriate parameters, and the sets should cover a broad range of strain rates. And, as is the case here, there may be multiple cases of data based on the same strain rate. After the conversion indicated in Figure 2(b) is made, the combined sets of converted data are plotted as shown in Figure 3. Converted Split Hopkinson Pressure Bar (SHPB) data for the Mock 900-21 material are also plotted in Figure 3 and are used along with the low rate data in the characterization procedure. The addition of the SHPB data broadens the response spectrum sufficiently to provide for a characterization covering expected engineering strain rates.

As can be observed in Figure 3, the resulting converted data cover a fairly wide range of logarithmic time decades. Now, considering only the first point in each converted data set, a new set of points is established, one that defines a log-log relationship between Young's modulus and relaxation time for the material system. Fitting this surface with a polynomial then provides that relationship in equation form.

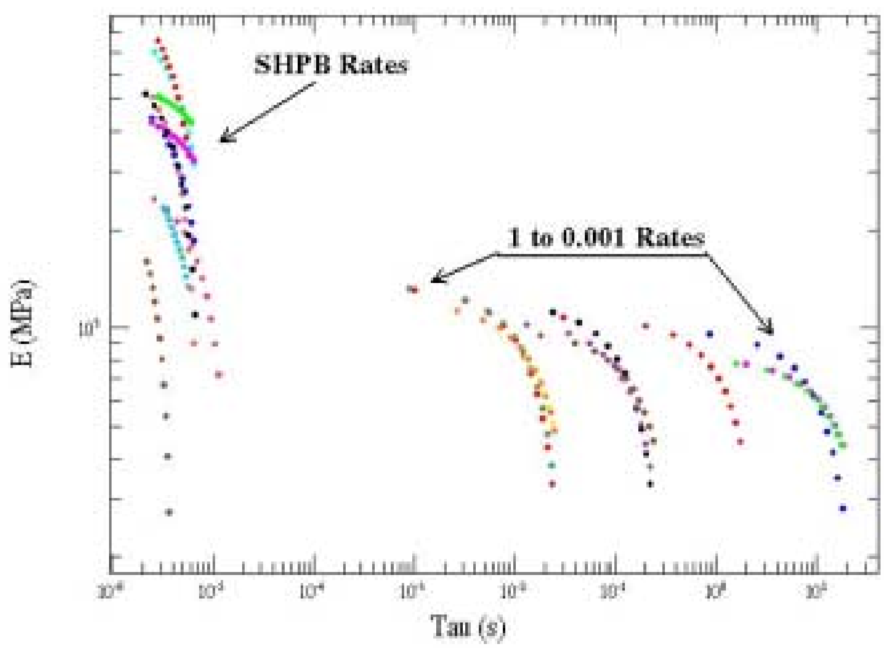


Figure 3. Log-log relationship between Young's modulus and relaxation time at different strain rates for Mock 900-21.

The next step is that of determining the number of Maxwell model elements to be used to characterize the viscoelastic response of the material system. This step will obviously depend in great measure upon the number of logarithmic time decrements represented in Figure 3.

To begin, extreme points of the logarithmic relaxation time domain to be considered are selected. In this case they are –6.0 and 3.0, representing a time domain of nine logarithmic decades. The $\log_{10}$ values of relaxation time are then selected to correspond to the mid-point of each of the nine logarithmic decades, and the two extreme points of the logarithmic time domain. The corresponding values of $\log_{10} E$ are calculated from the previously defined fitted log-log expression for the first-point values of Figure 3. This yields eleven values of $\log_{10} E$ and the eleven corresponding values of $\log_{10} \tau$. The values of E_n and τ_n are then obtained from these logarithmic terms, with $n = 1$ corresponding to the largest value of τ. Finally, the values G_n are calculated from the E_n values using a value of Poisson's ratio of, in this case, 0.30.

The viscoelasticity of the material system for which converted strain-rate data are shown in Figure 3 is thus characterized by eleven Maxwell model elements connected in parallel and having the following shear modulus and corresponding relaxation time values:

G_1 = 0.2547e+3 MPa τ_1 = 1.0e+3 s
G_2 = 0.2109e+0 MPa, τ_2 = 0.31623e+3 s
G_3 = 0.2353e+1 MPa, τ_3 = 0.31623e+2 s
G_4 = 0.8816e+1 MPa, τ_4 = 0.31623e+1 s
G_5 = 0.2089e+2 MPa, τ_5 = 0.31623e+0 s
G_6 = 0.4141e+2 MPa, τ_6 = 0.31623e-1 s
G_7 = 0.7730e+2 MPa, τ_7 = 0.31623e-2 s
G_8 = 0.1449e+3 MPa, τ_8 = 0.31623e-3 s
G_9 = 0.2846e+3 MPa, τ_9 = 0.31623e-4 s
G_{10} = 0.6058e+3 MPa, τ_{10} = 0.31623e-5 s
G_{11} = 0.5562e+3 MPa, τ_{11} = 1.0e-6 s

As noted earlier, the dashpot is removed from the first Maxwell element when the material model is employed in an analysis. Also, it can be noted that the initial shear modulus of the material system, G, is the sum of the G_n terms, in this case, 1997 MPa.

Internal Damage

The remaining step is that of determining the numerical values of the damage parameters to be incorporated into the material characterization procedure. The rate-dependent damage law is based upon statistical crack size growth mechanics per unit volume of material. The law governing the rate at which the average crack size per unit volume grows is a function of the effective strain rate. It requires that the instantaneous average crack growth rate be a function of the effective deviatoric strain rate, a scalar measure defined as a constant multiplying the square root of the inner product of the deviatoric strain-rate tensor with itself. The form of the damage law is

$$\log\left(KV_{\max}\right) = \log(\dot{e}deff)$$

where the multiplier K depends upon whether the stress state is tension or compression, $V_{\max}$ is a relatively complicated variable the value of which depends upon the value of the stress intensity factor in the statistical crack analysis development, and $\dot{e}deff$ is the effective deviatoric strain rate. Details of the statistical crack analysis development are found in Eqs. 24-31 in Hackett and Bennett (2000). An in-depth presentation of the formulation of the damage law is found in Bennett, Gray and Blumenthal (2001).

A physical interpretation of the damage law is that the local value of the damage variable is the total amount of growth of average-sized microcracks per unit volume that has occurred over the total number of average-sized microcracks that the material contained initially per unit volume. For example, an analysis might predict that the damage variable has reached a value of 10 mm. This value would indicate that N microcracks that began with a characteristic average radius of 1 mm would now have an average radius of 10 mm. This should not be interpreted to mean that a single microcrack is10 mm long, but rather that the total average crack growth is that value, with the average crack growth per unit volume being 10/N mm in size. Another way of stating this is that *the effect of the damage from microcrack extension* is 10 times the effect that the initial microcrack field has on the mechanical properties of the material.

A normalized damage parameter can be defined by the ratio of c, the current flaw size for N_o flaws per unit volume, to a, an average initial flaw size (Hackett and Bennett, 2000). The interpretation of this ratio leads to its usefulness in denoting the "remaining strength" of the material when its value has exceeded unity.

Employing the implicit finite element material model which was developed by Hackett and Bennett (2000), damage parameters which produce strain-rate simulations that correspond to actual strain-rate data for Mock 900-21 are determined by varying the damage law parameter to obtain the best fit to the stress-strain data in a trial and error manner. This is done for the same tests from which reduced viscoelastic data are obtained, as described earlier. Then, with the resulting data points that relate average crack velocity to effective strain rate, the relationship shown in Figure 4 can be developed.

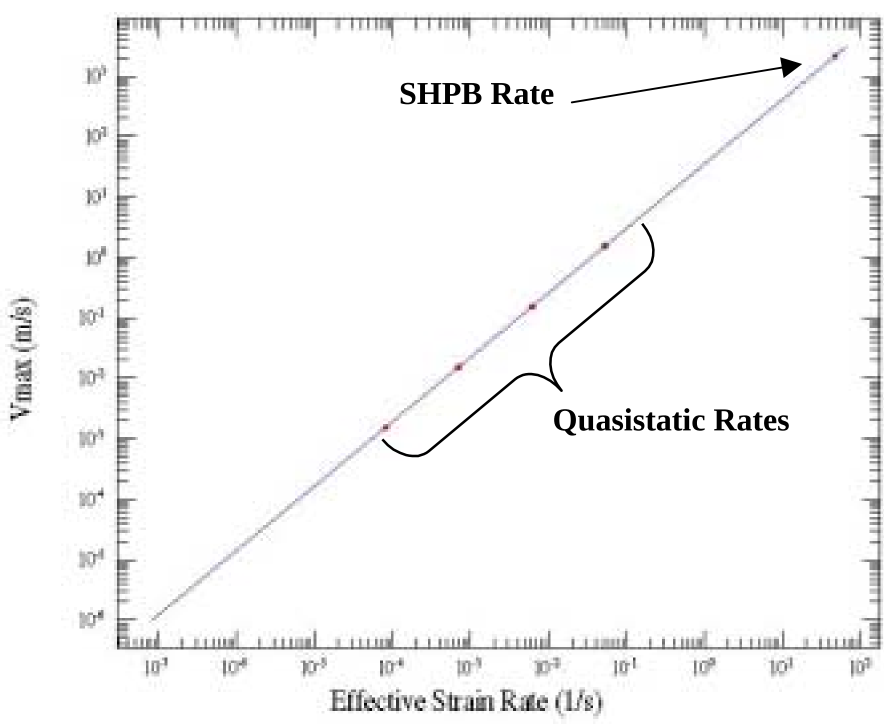


Figure 4. Graphical representation of damage law for Mock 900-21.

The resulting near-linear relationship can be noted. Thus, with the strain-rate response fully characterized, plots of stress versus damage parameter and stress versus strain can be developed. They are shown in Figures 5 and 6, respectively.

The interpretation of the "remaining strength" of the material as a function of the damage parameter arises naturally from Figure 5. At SHPB rates (not shown in this figure), the strain rates are generally high enough that the damage growth rate becomes limited to the maximum crack velocity in the material and the damage law parameter V_{max} at that value, along with the other fracture parameters, is set such that the material strength peaks at a value of c/a = 1. For lower and quasi-static rates, the maximum damage velocity, V_{max}, follows the experimentally determined best fit law, and for the growth rate as a function of the effective strain rate, the stress is "peaking", but has not reached a maximum value at a damage parameter of unity. As illustrated in Figure 5, the strength will peak at damage parameter values between 1.0 and 2.0 as the interaction between the viscoelastic effects and damage growth occurs.

The validity of the procedure can be attested to by a comparison of Figures 2(a) and 6.

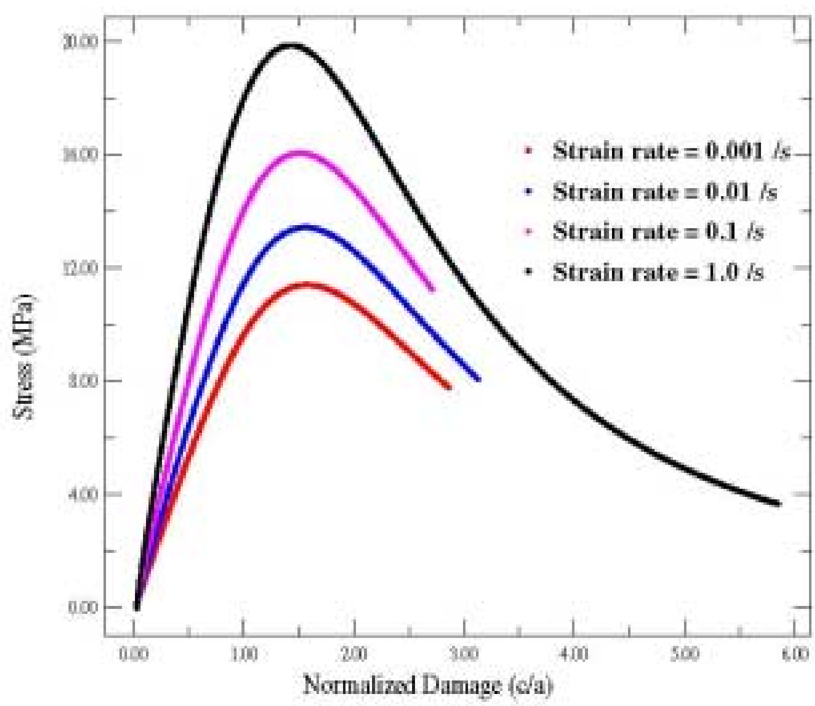


Figure 5. Relationship between uniaxial stress and damage parameter at different strain rates for Mock 900-21.

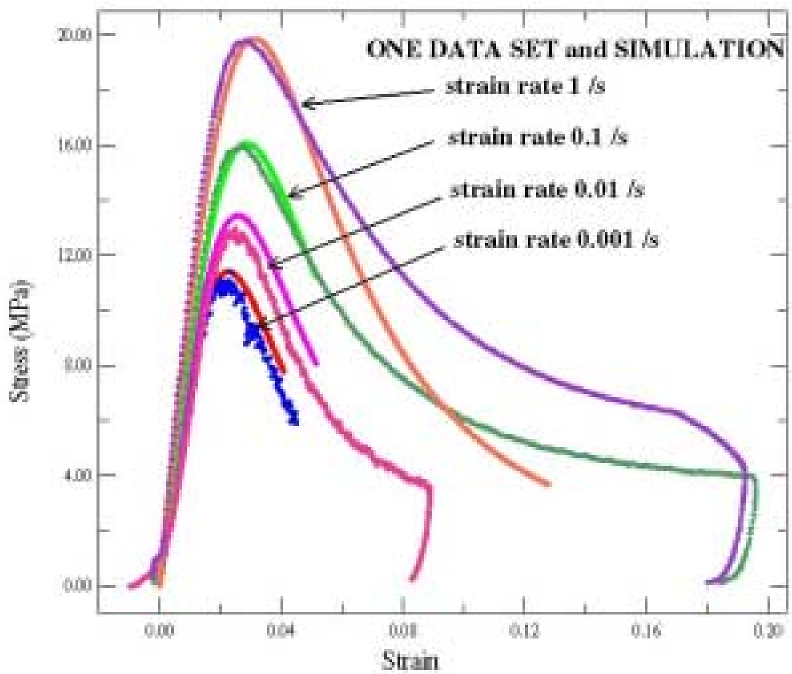


Figure 6. Simulated uniaxial stress-strain relationship for different strain rates for Mock 900-21

Conclusion

The method that is used to determine the viscoelasticity parameters for the model represented by Figure 1 has been illustrated using a set of data for a specific material known as Mock 900-21. By comparing Figures 2(a) and 6, one is able to ascertain the accuracy of the method. This type of model has been found to be generally applicable to materials that exhibit rate-dependent behavior and rate dependent damage, typically composites that have brittle particles embedded in a viscoelastic binder. The method has been used successfully to determine the parameters for two different energetic materials (PBX 9501, PBX 9502) at LANL, as well as for the simulant material Mock 900-21.

The continuum representation needed to capture the mechanical response for these types of materials is believed to be sufficiently accurate over ~10 decades in strain rate using this procedure.

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